## STAT 300 HW6 Question 1

1. The Ricci v. DeStefano case in New Haven, CT (129 S. Ct. 2659, Sup. Court, 2009), involved a claim of "reverse" discrimination. Firefighters in the city took examinations to progress through the ranks. One test was for promotion to lieutenant, and at the time the city had eight such positions to fill. The city's charter required the fire department to appoint from the candidates with the best ten scores on the relevant examination. All the top ten scores were from white applicants. The district declined to certify the exam and did not promote any of the candidates, on the grounds that doing so would fail to promote sufficient visible minority candidates to an existing position.
It is of interest to investigate whether there appears to be a difference in the mean scores on the examination for the three identified racial groups: white, black, and hispanic. Suppose the test score data were as displayed below:

|  | Blacks | Hispanics | Whites |
| :---: | :---: | :---: | :---: |
| Sample size: | 19 | 15 | 43 |
| Mean: |  |  |  |
| S.D.: |  |  |  |


(a) Taking that the underlying assumptions of ANOVA hold and that the approach will be applied, what is the estimate of the common variance of the test scores for the three racial groups? (Give your answer to two decimal places.)
(b) Complete the ANOVA table below, giving requested answers to two decimal places:

| Source of variation | df | SS | MS | F |
| :---: | :---: | :---: | :---: | :---: |
| Race | 2 | summary1\$Sum [2] |  | (ii) |
| Error | 74 |  | (i) |  |
| Total | 76 |  |  |  |

i. .
ii. .
(c) Which of the following inferences can be made when testing at the $5 \%$ significance level the null hypothesis that the racial groups have the same mean test scores?
i. Since the observed F statistic is greater than the 95 th percentile of the $F_{2,74}$ distribution we can reject the null hypothesis that the three racial groups have the same mean test score.
ii. Since the observed F statistic is less than the 95 th percentile the $F_{2,74}$ distribution we can reject the null hypothesis that the three racial groups have the same mean test score.
iii. Since the observed F statistic is greater than the 95th percentile of the $F_{2,74}$ distribution we do not reject the null hypothesis that the three racial groups have the same mean test score.
iv. Since the observed F statistic is less than the 95 th percentile of the $F_{2,74}$ distribution we do not reject the null hypothesis that the three racial groups have the same mean test score.
v. Since the observed F statistic is greater than the 5th percentile of the $F_{2,74}$ distribution we do not reject the null hypothesis that the three racial groups have the same mean test score.
vi. Since the observed F statistic is less than the 5 th percentile of the $F_{2,74}$ distribution we do not reject the null hypothesis that the three racial groups have the same mean test score.
(d) Suppose we perform our pairwise comparisons, to test for a significant difference in the mean scores between each pair of racial groups. If investigating for a significant difference in the mean scores between blacks and whites, what would be the smallest absolute distance between the sample means that would suggest a significant difference? Assume the test is at the $5 \%$ significance level, and give your answer to 3 d.p.
The following information is not visible to a student.
Randomisation: The following are used for generating the data and the boxplot
$S 1<-\operatorname{abs}(\operatorname{round}($ rnorm $(19,63.7,8), 1))$

```
S2<- abs(round(rnorm(15,63.6,5.8),1))
S3<- abs(round(rnorm(43,71.8,9),1))
boxplot(S1,S2,S3, range = 1.5, names=c("Blacks","Hispanics","Whites"),
xlab = "Racial Group", ylab = "Lieutenant exam scores",
main = "Hypothetical Lieutenant Exam Scores by Racial Group")
Summary statistics for the table can be found via, for example
round(mean(S1),2)
round(sd(S1),2)
The ANOVA table can be created as follows
race <- as.factor(rep(1:3,c(19,15,43)))
model1 <- aov(c(S1,S2,S3) ~race)
summary1 <- anova(model1)
From the above object the relevant statistics can be extracted as
follows:
summary1$Df
summary1$Sum
summary1$Mean
summary1$F
The above give the degrees of freedom, the sums of squares, the mean squares and the \(F\) statistic respectively. All of these are vectors, so for example, the error \(S S\) is taken via summary1\$Sum [2]
The total \(S S\) is found by adding the two sums of squares.
Attempts: Suggest four attempts should be permitted, with any incorrect parts indicated to students after each attempt.
Solution: Available in WeBWorK
Tagging: Statistical inference; Analysis of variance; estimate the common unknown variance using the error (i.e., residual) mean square, compute statistics for the ANOVA table, draw inference from an ANOVA table, compute significant difference for a multiple comparison test.
DBsubject('Statistics')
DBchapter('Hypothesis Testing')
DBsection('One-way ANOVA')
Level('5')
```



